Fanno Flow - NUMERICALS

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Points to Remember

1) $h_{01} = h_{02} \Rightarrow T_{01} = T_{02}$

2) $P_{01} \neq P_{02}$

3) p^* & T^* { Conditions at $M = 1$ } are used to relate between (1) & (2)

4) To find P_0, T_0 { stagnation Condition } use isentropic tables & Mach number at the required point

5) $P = \rho R T$, $M_1 = \dfrac{C_1}{\sqrt{\gamma R T_1}}$, $M_2 = \dfrac{C_2}{\sqrt{\gamma R T_2}}$

6) Value of friction factor / friction coefficient / Fanning friction factor indicates a numerical problem based on Fanno Flow. { It will be given for isothermal flow numerical also - there it will be specified isothermal flow or temperature or static enthalpy remains constant }

7) We need Mach number at a point to get properties at that point so finding that is very important.

8) Gas table & scientific Calculator are of great importance.

Q) A circular duct passes 10 kg/s of air at an exit Mach number of 0.5. The Mach number at entry is 0.15. The entry pressure & temperature are 3.5 bar & 40°C. The coefficient of friction is 0.009.

Determine a) diameter of duct b) Length of duct c) pressure & temperature at exit d) Stagnation pressure loss

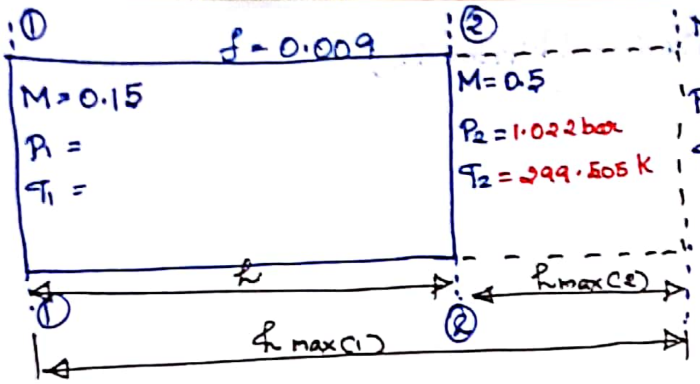
Sol: Given

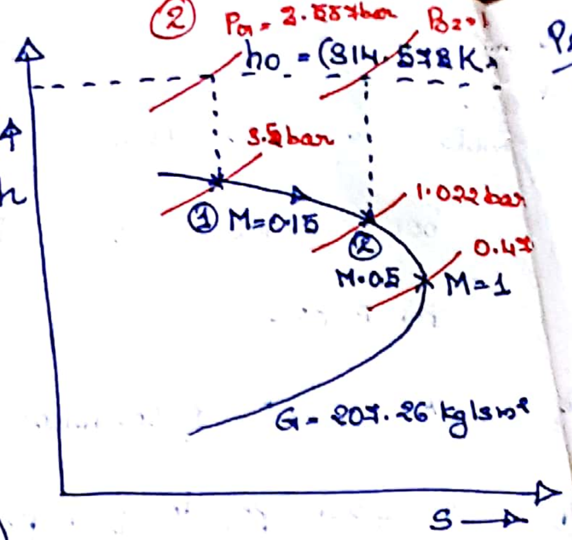
$\dot{m} = \dot{m}_1 = \dot{m}_2 = 10 \text{ kg/s}$, $T_1 = 40°C = 313 \text{ K}$

$M_1 = 0.15$, $\bar{f} = 0.009$

$M_2 = 0.50$, $\gamma = 1.4$, $R = 287 \text{ J/kg K}$ { nothing specified }

$P_1 = 3.5 \text{ bar}$





$\dot{m}_1 = \dot{m}_2 = 10\ kg/s$

$\dot{m}_1 = \rho_1 A C_1 = \left(\frac{P_1}{RT_1}\right) \times \left(\frac{\pi}{4} d_1^2\right) \times \left(M_1 \times \sqrt{\gamma R T_1}\right)$

$$10\ kg/s = \frac{3.5 \times 10^5}{287 \times 313} \times \frac{\pi}{4} \times d_1^2 \times 0.15 \times \sqrt{(1.4 \times 287 \times 313)}$$

$$\boxed{d = 0.248\ m}$$

From Fanno tables for $\gamma = 1.4$

	M	P/P^*	T/T^*	P_0/P_0^*	$\frac{4\bar{f} L_{max}}{D}$	
inlet 1	0.15	7.309	1.1945	3.928	28.854	{interpolate}
exit 2	0.5	2.188	1.143	1.340	1.069	

$L = L_{max(1)} - L_{(max)(2)}$

$$L = \left[\frac{4\bar{f} L_{max(1)}}{D} - \frac{4\bar{f} L_{max(2)}}{D}\right] \times \frac{D}{4\bar{f}}$$

$$= (28.854 - 1.069) \times \frac{0.248}{4 \times 0.009} = \boxed{187.963\ m}$$

$\frac{P_1}{P^*} = 7.309 \Rightarrow P^* = \frac{P_1}{7.309} = \frac{3.5}{7.309} = 0.479\ bar$

$\frac{T_1}{T^*} = 1.194 \Rightarrow T^* = \frac{T_1}{1.194} = \frac{313}{1.194} = 262.034\ K$

$$\frac{P_2}{P^*} = 2.188 \Rightarrow P_2 = 2.188 \times P^* = 2.188 \times 0.478 = \boxed{1.022 \text{ bar}}$$

$$\frac{T_2}{T^*} = 1.143 \Rightarrow T_2 = 1.148 \times T^* = 1.148 \times 262.084 = \boxed{299.505 \text{ K}}$$

To find stagnation pressure loss we need stagnation pressure values at inlet ($M = 0.15$) & outlet ($M = 0.5$) for which we will use gas tables.

For $\gamma = 1.4$ from Isentropic tables

	M	T/T_0	P/P_0
Inlet ①	0.15	0.995	0.984
outlet ②	0.50	0.952	0.843

$$\frac{T_1}{T_{01}} = 0.995 \Rightarrow T_0 = \frac{T_1}{0.995} = \frac{313}{0.995} = 314.578\text{ K} = T_{02}$$

$$\frac{P_1}{P_{01}} = 0.984 \Rightarrow P_{01} = \frac{P_1}{0.984} = \frac{3.5}{0.984} = 3.557 \text{ bar}$$

$$\frac{P_2}{P_{02}} = 0.843 \Rightarrow P_{02} = \frac{P_2}{0.843} = \frac{1.022}{0.843} = 1.212 \text{ bar.}$$

$$\Delta P_0 = P_{01} - P_{02} = 3.557 - 1.212 = \boxed{2.345 \text{ bar}}$$

Q) A gas ($\gamma = 1.3$, $R = 287$ J/kgK) at $p_1 = 1.0$ bar, $T_1 = 400$ K enters inside 30 cm diameter duct at a Mach number of 2.0. A normal shock occurs at a Mach number of 1.5 & the exit Mach number is 1.0. If shock friction ha... mean value of friction factor is 0.003. Determine

a) Length of duct upstream of shock & down stream of shock.

b) mass flow rate, c) Change in entropy upstream, across & down-stream of shock.

Sol:

We have a constant area duct with normal shock and friction factor is given so we have to consider the flow upstream & downstream has simple friction flow rather than isentropic flow.

Inlet of duct ① Condition just upstream of shock ⓧ

outlet of duct ② Conditions just downstream of shock ⓨ

We consider ⓧ & ⓨ just upstream & downstream 'cox the flow proceeds property variation occurs due to frictions & condition obtained at ⓧ & ⓨ will change (in shock numericals earlier it was isentropic flow through constant area so no change hence ⓧ & ⓨ conditions remained same through out)

Given

$\gamma = 1.3$

$R = 287$ J/kgK

$p_1 = 1.0$ bar

$T_1 = 400$ K

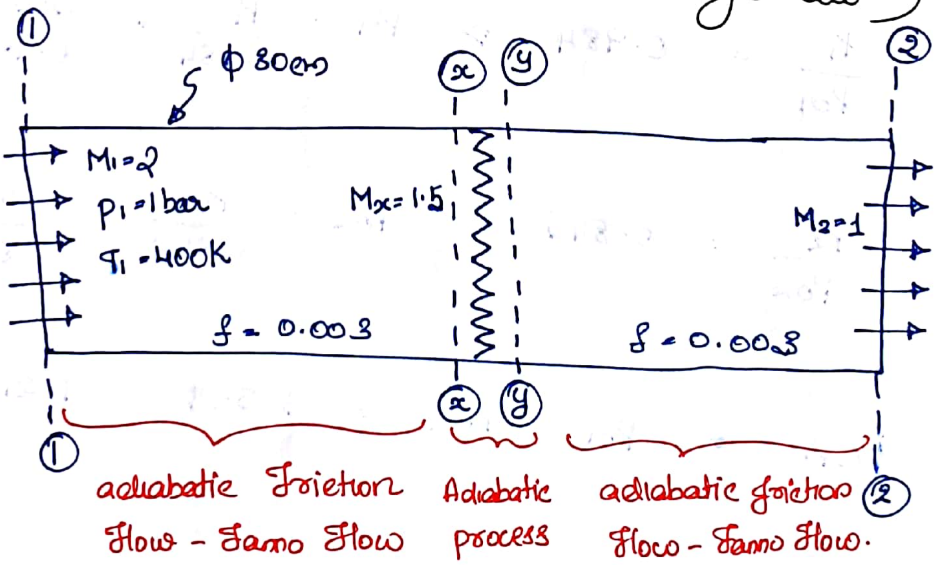
$d = 30$ cm $= 0.3$ m

$M_1 = 2.0$

$M_x = 1.5$

$M_2 = 1.0$

$\bar{f} = 0.003$

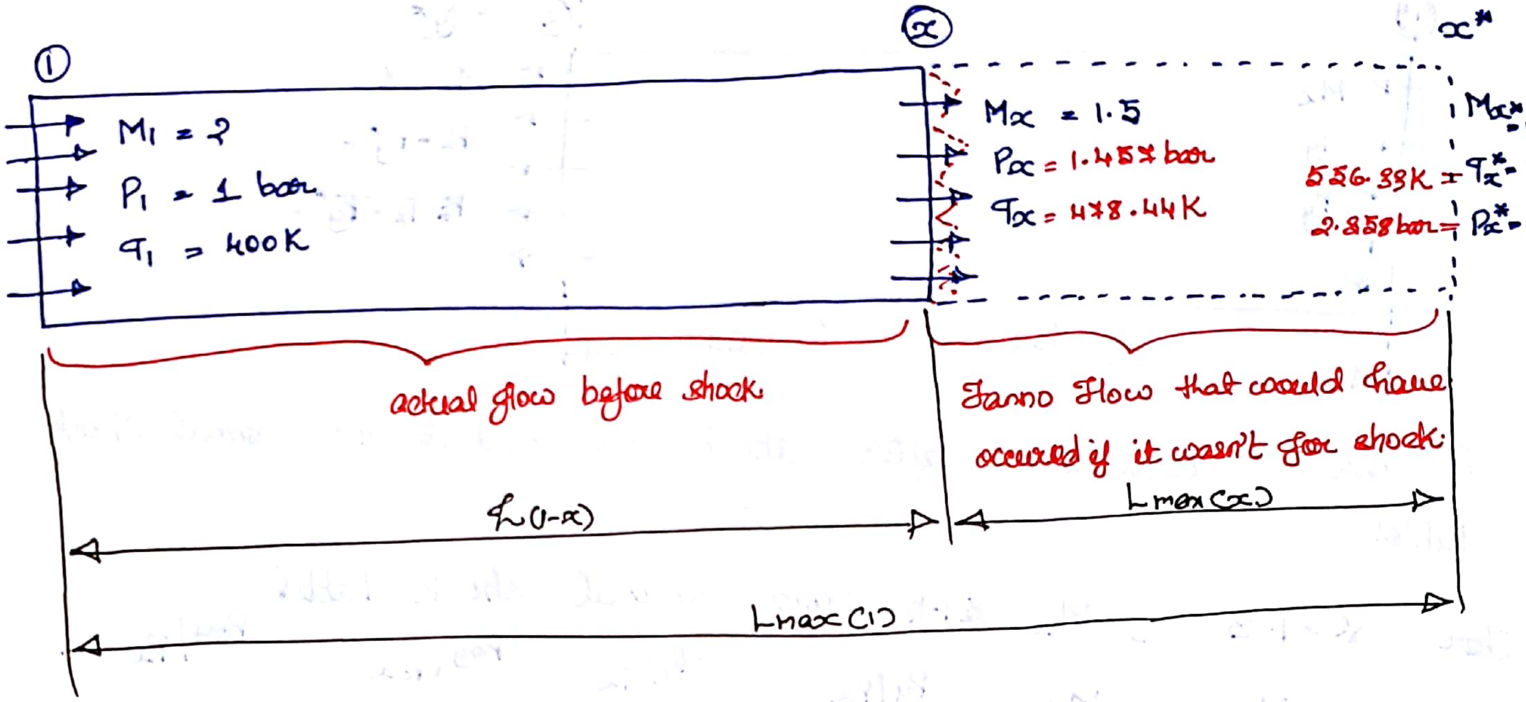


The normal shock divides the entire flow into two Fanno Flow a) ① → ⓧ b) ⓨ → ②

So both will have separate points of critical. Conditions

In this problem is downstream, $M = 1$ at exit will be the critical condition for downstream Fanno flow

Consider the flow upstream of shock i.e. (5) from (1) — (x). If the shock had not occurred it would have been a simple fanno flow & friction decelerates supersonic flow. Here if the flow had continued from (x) this fanno flow in downstream there would be a point where "M=1". This is our critical condition for upstream flow. (x^*)



For $\gamma = 1.3$, from Fanno flow tables

	M	P/P^*	C/C^*	$\frac{T}{T^*}$	$\frac{P_0}{P_0^*}$	$\frac{4\bar{f}L_{max}}{D}$
inlet of duct (1)	2	0.424	1.696	0.719	1.783	0.357
upstream of shock (x)	1.5	0.618	1.391	0.860	1.189	0.156

Length of duct upstream

$$L_{(1-x)} = L_{max(1)} - L_{max(x)}$$

$$= \left[\frac{4\bar{f}L_{max(1)}}{D} - \frac{4\bar{f}L_{max(x)}}{D}\right] \times \frac{D}{4\times\bar{f}}$$

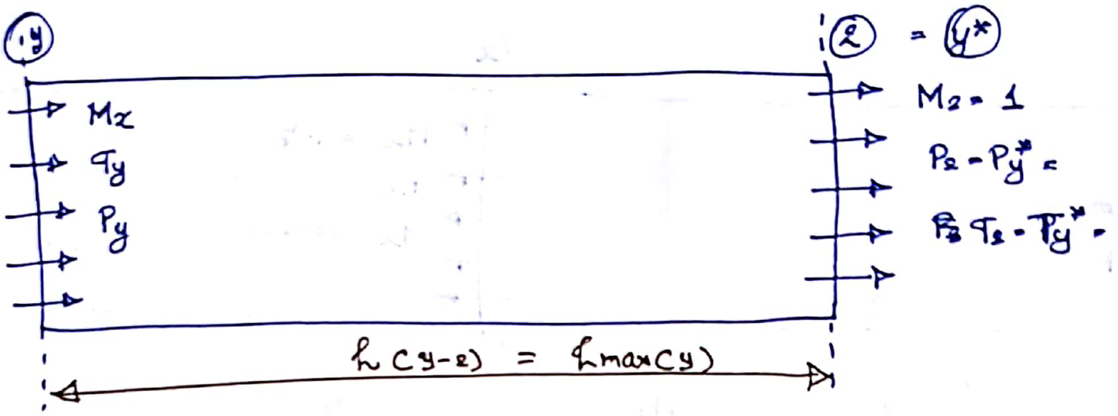
$$L_{(1-x)} = \left[0.357 - 0.156\right] \times \frac{0.20}{4\times 0.003} = 5.025\ m$$

$$\frac{P_1}{P_x^*} = 0.424 \Rightarrow P_x^* = \frac{1.0}{0.424} \doteq 2.358\ bar$$

$$\frac{T_1}{T_x^*} = 0.719 \Rightarrow T_x^* = \frac{400}{0.719} = 556.328\ K$$

$$\frac{P_x}{P_x^*} = 0.618 \Rightarrow P_x = 0.618 \times P_x^*$$
$$= 0.618 \times 2.358 = 1.457 \text{ bar}$$

$$\frac{T_x}{T_x^*} = 0.860 \Rightarrow T_x = 0.86 \times T_x^*$$
$$= 0.86 \times 556.328 = 478.44 \text{K}$$



To obtain conditions after shock we need to use normal shock tables

For $\gamma = 1.3$, $M_x = 1.5$ from normal shock tables

M_x	M_y	P_y/P_x	T_y/T_x	P_{oy}/P_{ox}	P_{oy}/P_x
1.5	0.694	2.413	1.247	0.926	3.265

$M_y = 0.694$

$$\frac{P_y}{P_x} = 2.413 \Rightarrow P_y = 2.413 \times P_x = 2.413 \times 1.457 = 3.516 \text{ bar}$$

$$\frac{T_y}{T_x} = 1.247 \Rightarrow T_y = 1.247 \times T_x = 1.247 \times 478.44 = 596.615 \text{K}$$

We will use this to determine downstream conditions

For $\gamma = 1.3$, $M_y = 0.694 \approx 0.69$ from Fanno tables

M_y	P_y/P_y^*	T_y/P_y^*	$\frac{4\bar{f}L_{max}(y)}{D}$
0.69	1.502	1.078	0.254

Length of duct downstream $L(y-2) = L_{max}(y)$

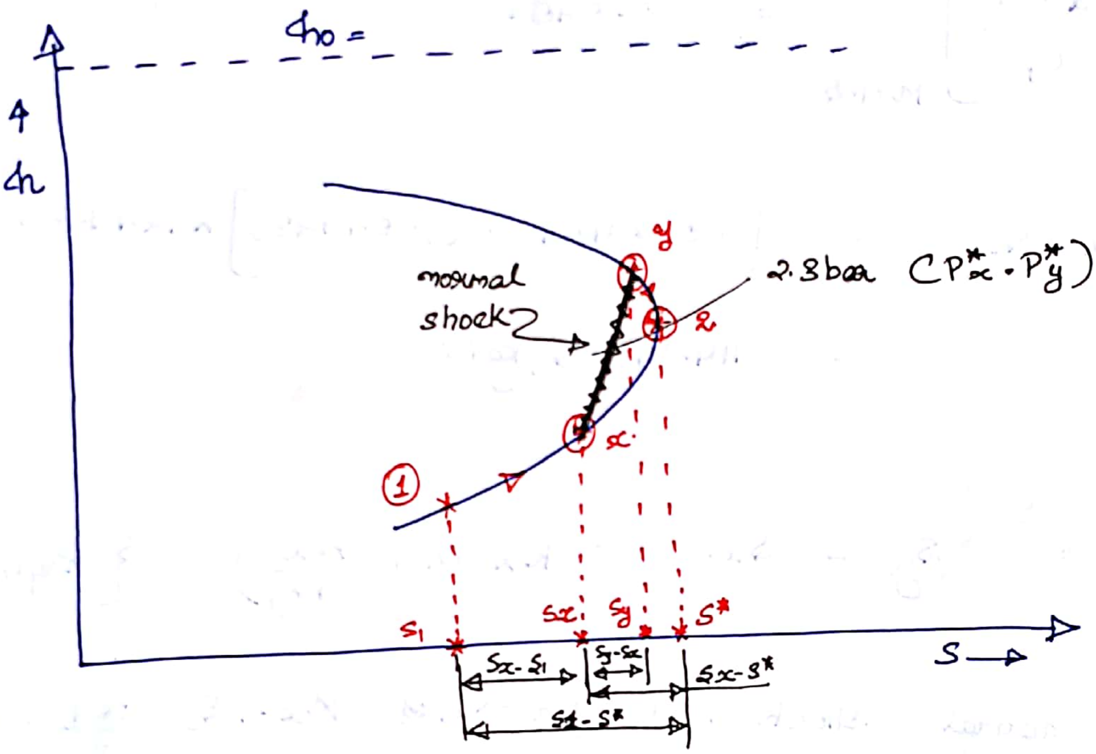
$$= \left[\frac{4\bar{f}L_{max}(y)}{D}\right] \times \frac{D}{4 \times \bar{f}}$$

the critical conditions

$$L_{(y-2)} = 0.254 \times \frac{0.8}{4 \times 0.008} = 6.888\ m.$$

$$\frac{P_y}{P_y^*} = 1.502 \Rightarrow P_y^* = \frac{P_y}{1.502} = \frac{3.516}{1.502} = 2.341\ bar$$

$$\frac{T_y}{T_y^*} = 1.078 \Rightarrow T_y^* = \frac{T_y}{1.078} = \frac{596.615}{1.078} = 556.025\ K$$



mass flow rate $\dot{m} = \dot{m}_1 = \dot{m}_x = \dot{m}_y = \dot{m}_2$

$$\dot{m} = \rho_1 A c_1 = \frac{P_1}{RT_1} \times \frac{\pi}{4} d_1^2 \times c_1 \sqrt{\gamma R T_1}$$

$$\dot{m} = \frac{1 \times 10^5}{287 \times 400} \times \frac{\pi}{4} \times (0.8)^2 \times 2\sqrt{1.4 \times 287 \times 400} = 47.57\ kg/s$$

entropy change upstream of shock $\Delta S_{(x-1)}$
$$= S_x - S_1$$
$$= S_x - S^* - S_1 + S^*$$
$$= (S_x - S^*) - (S_1 - S^*)$$
$$= \left[\left(\frac{S - S^*}{c_p}\right)_x - \left(\frac{S - S^*}{c_p}\right)_1\right] \times c_p$$

$$\frac{s_x - s^*}{c_p} = \ln\left[M^2 \left\{ \frac{\gamma+1}{2M^2\left(1+\frac{\gamma-1}{2}M^2\right)} \right\}^{\frac{\gamma+1}{2\gamma}} \right]$$ (Page 8, Eq

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$$\left[\frac{s_1 - s^*}{c_p}\right]_{M=2} = -0.1828$$

$$c_p = \frac{1.3 \times 287}{1.3-1} = 1243.67 \text{ J/kgK}$$

$$\left[\frac{s_x - s^*}{c_p}\right]_{M=1.5} = -0.0404$$

$$\Delta s_{1x} = s_1 - s_x = \left[-0.0404 - (-0.1828)\right] \times 1243.67$$

$$= 114.915 \text{ J/kgK}$$

$$\Delta s_{(y-x)} = s_y - s_x = R \times \ln\left(\frac{P_{0x}}{P_{0y}}\right)$$ { Page 5, Eq 4.10 }

From normal shock table for $\gamma = 1.3$, $M_x = 1.5$, $\frac{P_{0y}}{P_{0x}} = 0.926$

$$\Delta s_{(y-x)} = 287 \times \ln\left(\frac{1}{0.926}\right) = \cancel{309.935}\ \text{J/kgK}$$ 22.06 J/kgK

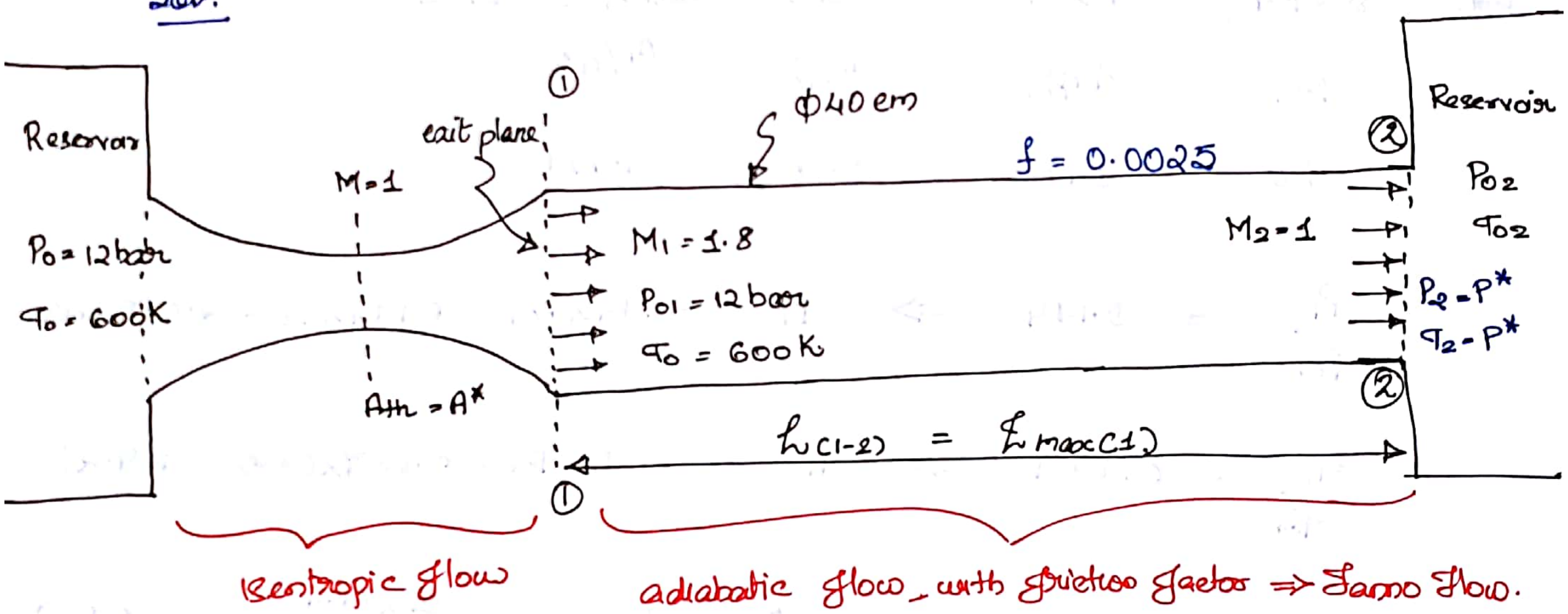
For down stream change in entropy

$$\Delta s_{(2-y)} = s_2 - s_y = s^* - s_y = -(s_y - s^*) = -\left(\frac{s_y - s^*}{c_p}\right)c_p$$

$$\Delta s_{(2-y)} = -\left[-0.0221\right] \times 1243.67 = 27.48 \text{ J/kgK}$$

Eg 8 A convergent - divergent nozzle is provided with a pipe (a) of constant area at its exit. The exit diameter of nozzle & that of pipe is 40 cm. The mean coefficient of friction for pipe is 0.0025. Stagnation pressure & temperature of air is 12 bar & 600K. The flow is isentropic in nozzle & adiabatic in pipe. Mach number at entry & exit of pipe are 1.8 & 1 respectively. Determine a) Length of pipe b) diameter of nozzle throat c) Pressure & temperature at pipe exit d) If pipe discharges into a reservoir at exit, what is pressure & temperature in exit reservoir. e) Depict variation of static & stagnation pressure throughout the flow process.

Sol:



The exit condition of nozzle is same has inlet condition of duct. Reservoir conditions refers to stagnation condition. If flow accelerated from stagnation condition to $M=1.8$, then Mach number is unity at throat.

Therefore $A_{th} = A^*$

From isentropic tables we will get A/A^* of which A stands for area of nozzle exit.

For $\gamma = 1.4$, $M = 1.8$ from Fanno flow tables 10 — from

M_1	P_1/P^*	T_1/T^*	P_0/P_0^*	$\frac{4\bar{f}L_{max(1)}}{D}$
1.8	0.474	0.728	1.439	0.242

$$L_{(1-2)} = L_{max(1)} = \left[\frac{4\bar{f}L_{max}}{D}\right] \times \frac{D}{4\bar{f}}$$

$$= 0.242 \times \frac{0.4}{4 \times 0.0025} = 9.68\,m$$

For $\gamma = 1.4$, $M = 1.8$ from Isentropic Tables

M_1	P_1/P_{01}	T_1/T_{01}	A_1/A^*
1.8	0.174	0.607	1.439

$$\frac{P_1}{P_{01}} = 0.174 \Rightarrow P_1 = 0.174 \times P_{01} = 0.174 \times 12 = 2.088\ bar$$

$$\frac{T_1}{T_{01}} = 0.607 \Rightarrow T_1 = 0.607 \times T_{01} = 0.607 \times 600 = 364.2\,K$$

$$\frac{A_1}{A^*} = 1.439 \Rightarrow A^* = \frac{A_1}{1.439} = \frac{\frac{\pi}{4} \times (0.4)^2}{1.439} = 0.087\,m^2$$

$$A^* = A_{th} = \frac{\pi}{4} d_{th}^2 \Rightarrow d_{th} = \sqrt{\frac{4 \times A_{th}}{\pi}} = 0.333\,m$$

From Fanno Tables $\frac{P_1}{P^*} = 0.474 \Rightarrow P^* = \frac{P_1}{0.474} = \frac{2.088\ bar}{0.474}$

$$P^* = P_2 = 4.405\ bar.$$

$$\frac{T_1}{T^*} = 0.728 \Rightarrow T^* = \frac{T_1}{0.728} = \frac{364.2\,K}{0.728}$$

$$T_2 = T^* = 500.27\,K$$

From ~~isentropic~~ Fanno tables $\frac{P_{01}}{P_0^*} = 1.439$

$$P_{02} = P_0^* = \frac{P_{01}}{1.439} = \frac{12\text{ bar}}{1.439} = 8.339\text{ bar}$$

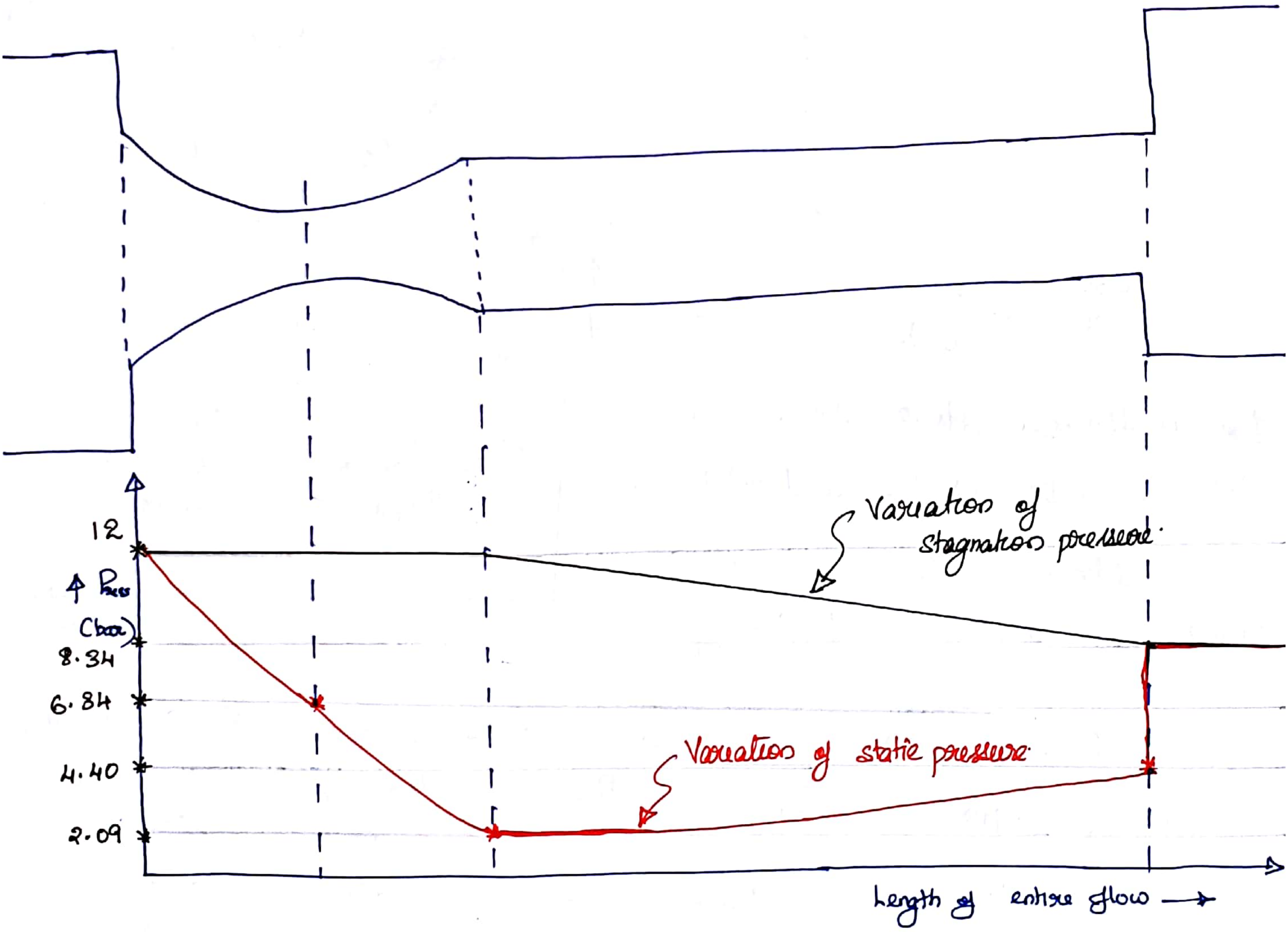
Stagnation Temperature remains same through out

$$\therefore\ T_{01} = T_{02} = 600K.$$

At the throat $M = 1$ & flow is isentropic $\therefore$ for $\gamma = 1.4$, $M = 1$.

from isentropic tables.

$$\frac{P_{th}}{P_0} = 0.528 \Rightarrow P_{th} = 0.528 \times P_0$$
$$= 0.528 \times 12 = 6.336\text{ bar}.$$



8) Air enters a long circular duct (d = 12.5cm, f = 0.0045) at a Mach number of 0.5, pressure 3 bar & temperature 312K. If the flow is isothermal throughout the duct, determine
a) Length of duct required to change the Mach number to 0.7.
b) Pressure & temperature of air at M = 0.7.
c) Length of pipe required to attain limiting Mach number
d) State of air at limiting Mach number.

Sol:

Given : $d_1 = 12.5cm = 0.125m$

$f = 0.0045$

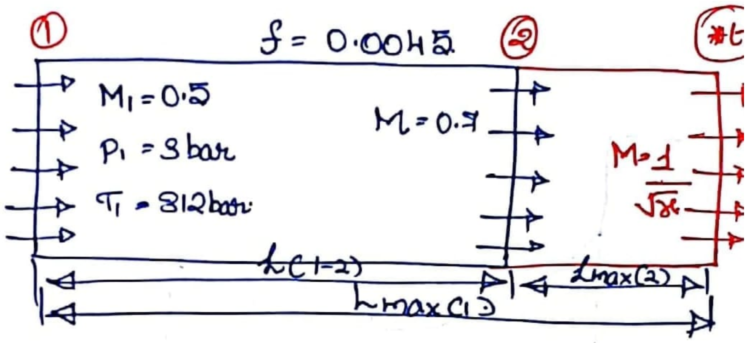
$M_1 = 0.5$

$M_2 = 0.7$

$P_1 = 3$ bar

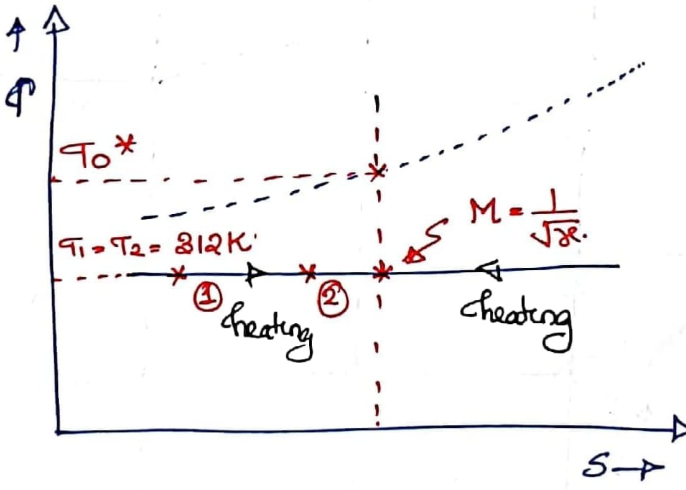
$T_1 = 312K$

not specified. $\begin{cases} \gamma = 1.4 \\ R = 287 \text{ J/kgK} \end{cases}$



For isothermal flow the limiting Mach number is $\frac{1}{\sqrt{\gamma}}$ and not M = 1.

The conditions of heating & cooling change on either side of $M = \frac{1}{\sqrt{\gamma}}$



For $\gamma = 1.4$ from isothermal flow with friction tables (Page 99)

M	P/p^{*t}	T_0/T_0^{*t}	P_0/P_0^{*t}	$\frac{4fL_{max}}{D}$	$\frac{c}{c^{*t}}$
0.5	1.690	0.918	1.256	0.807	0.592
0.7	1.207	0.961	1.049	0.0808	0.828

Length of duct $L_{(1-2)}$

$$L_{(1-2)} = L_{max(1)} - L_{max(2)}$$

$$= \left[\frac{4\bar{f}L_{max(1)}}{D} - \frac{4\bar{f}L_{max(2)}}{D}\right] \times \frac{D}{4\bar{f}}$$

$$= (0.807 - 0.0808) \times \frac{0.125}{4 \times 0.0045} = \boxed{5.048\ m}$$

From gas tables $\frac{P_1}{P^{*t}} = 1.69 \Rightarrow P^{*t} = \frac{P_1}{1.69} = \frac{3}{1.69} = 1.775$ bar

$$\frac{P_2}{P^{*t}} = 1.207 \Rightarrow P_2 = 1.207 \times P^{*t} = 1.207 \times 1.775$$

$$\boxed{P_2 = 2.142}$$

$$\boxed{T_2 = T_1 = 312 K.}$$ { isothermal flow }

Length of duct required to obtain limiting Mach number $= L_{max(1)}$

$$\frac{4\bar{f}L_{max(1)}}{D} = 0.807.$$

$$L_{max(1)} = \frac{0.807 \times D}{4 \times 0.0045} = \frac{0.807 \times 0.125}{4 \times 0.0045} = \boxed{5.604\ m}$$

$$\frac{C_1}{C^{*t}} = 0.592 \Rightarrow C^{*t} = \frac{C_1}{0.592} = \frac{M_1 \times \sqrt{\gamma R T_1}}{0.592}$$

$$C^{*t} = \left(0.5 \times \sqrt{1.4 \times 287 \times 310}\right) / 0.592 = 298.081\ m/s.$$

$$\boxed{\begin{aligned} P^{*t} &= 1.775\ bar \\ C^{*t} &= 298.081\ m/s \\ T^{*t} &= T_1 = T_2 = 310\ K \end{aligned}}$$ (Isothermal flow)